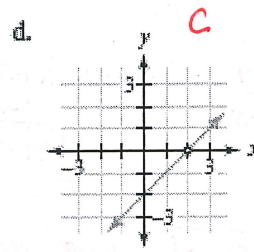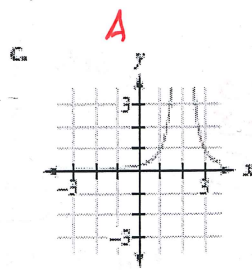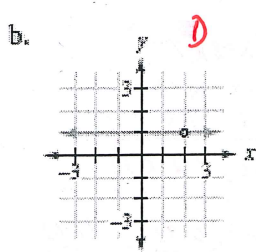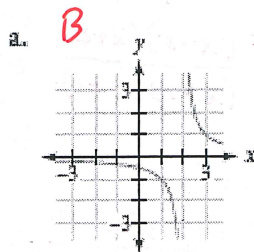*Objectives:*

1. Identify characteristics of the graph of a rational function from its equation.
2. Learn to write the equation of a rational function from its graph.
3. Rewrite a function as a rational function.
4. Review factoring.
5. Identify *holes*, *vertical asymptotes*, *x-intercepts*, *y-intercepts*, and *horizontal asymptotes* [push] of rational functions.

Hole: A ___*undefined*___ point in the graph of a relation.

This open dot represents a hole where a point is missing.

Predicting <u>Asymptotes</u> and <u>Holes</u>

<u>Step 1</u>: Match each rational function with a graph. Investigate each graph. Describe the unusual occurrences at exactly $x = 2$ and other values nearby.

a. **B**

b. **D**

c. **A**

d. **C**

A. $y = \dfrac{1}{(x-2)^2}$ B. $y = \dfrac{1}{x-2}$ C. $y = \dfrac{(x-2)^2}{x-2}$ D. $y = \dfrac{x-2}{x-2}$

ALL Y-VALUES MUST BE POSITIVE

$y = \dfrac{1}{(1-2)^2} = \dfrac{1}{(-1)^2} = 1$

$y = \dfrac{1}{(0-2)^2} = \dfrac{1}{(-2)^2} = \dfrac{1}{4}$

$y = \dfrac{1}{1-2} = \dfrac{1}{-1} = -1$

$y = \dfrac{1}{0-2} = -\dfrac{1}{2}$

$y = x - 2$

$y = 1$

Characteristic	Explain how to determine these charaeteristics from the graph's equation.	Give an example.
Vertical Asymptote	Set the denominator equal to zero and solve for x. Values of x that make denominator 0.	$y = \dfrac{1}{x+5}$ $x = -5$
X- Intercepts	Put 0 in for y and solve for x. Values of x that make numerator 0	$y = \dfrac{(x-1)(x+5)}{(x+2)}$ $x-1=0 \quad x+5=0$ $x=1 \quad x=-5$ $(1,0) \quad (-5,0)$
Y – Intercepts	Put 0 in for x and solve for y. y-values when x=0	$y = \dfrac{(x-1)(x+5)}{x+2}$ $y = \dfrac{(0-1)(0+5)}{(0+2)}$ $= \dfrac{(-1)(5)}{2}$ $= -\dfrac{5}{2} \quad (0, -\dfrac{5}{2})$
Holes	When we have the same factor in the numerator and denominator The x-value that makes the factor equal 0.	$y = \dfrac{(x+4)(x-3)}{(x+4)}$ $x+4=0$ $x = -4$ Hole when $x = -4$

Example 1: Find any *x*-intercepts, *y*-intercept, vertical asymptotes, and holes in the rational functions below:

a. $y = \dfrac{x-4}{x+5}$

x-int: $\dfrac{0}{1} = \dfrac{x-4}{x+5}$
(y=0)

$x-4=0$
$+4 \quad +4$
$x=4 \quad \boxed{(4,0)}$

y-int: $y = \dfrac{0-4}{0+5}$
(x=0)

$y = -\dfrac{4}{5} \quad \boxed{(0, -\dfrac{4}{5})}$

VA: $x+5=0$
$-5 \quad -5$
$\boxed{x=-5}$

HOLES: NONE

b. $y = \dfrac{(x+8)(x-2)}{(x+8)(x+1)}$

x-int: $0 = (x+8)(x-2)$
(y=0) $x+8=0 \quad x-2=0$
$-8 \, -8 \quad +2 \, +2$
$\boxed{(-8,0) \quad (2,0)}$

y-int $y = \dfrac{(0+8)(0-2)}{(0+8)(0+1)} = \dfrac{8(-2)}{8}$
(x=0)
$y = -2 \quad \boxed{(0,-2)}$

VA: $(x+8)(x+1) = 0$
$x+8=0 \quad x+1=0$
$-8 \, -8 \quad -1 \, -1$
$\boxed{x=-8} \quad \boxed{x=-1}$

HOLE: $x+8=0$
$-8 \, -8$
$\boxed{HOLE \ WHEN \ x=-8}$

c. $y = \dfrac{x^2 - 3x - 10}{x^2 - 5x - 14}$

$y = \dfrac{(x-5)(x+2)}{(x-7)(x+2)}$

x-int (y=0):
$0 = (x-5)(x+2)$
$x-5=0 \quad x+2=0$
$\boxed{(5,0) \quad (-2,0)}$

y-int (x=0)
$y = \dfrac{0^2 - 3(0) - 10}{0^2 - 5(0) - 14} = \dfrac{-10}{-14} = \dfrac{5}{7}$
$\boxed{(0, \dfrac{5}{7})}$

VA: $x-7=0 \quad x+2=0$
$+7 \, +7 \quad -2 \, -2$
$\boxed{x=7} \quad \boxed{x=-2}$
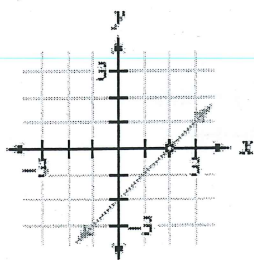
HoLE: $x+2=0$
$-2 \, -2$
$x = -2$
$\boxed{HOLE \ WHEN \ x = -8}$

d.

Reexamine the graph and its matching equation at the left. What characteristic do you see on the graph?

There is a hole at x=2

Why do you notice about the equation that would cause the hole to override the vertical asymptote?

More factors on top

c. $y = \dfrac{(x-2)^2}{x-2}$

$y = \dfrac{(x-2)(x-2)}{(x-2)}$ $y = x-2$ ← *Same except x still can't equal 2.*

Example 2: Find any x-intercepts, y-intercept, vertical asymptotes, and holes in the rational functions below:

a. $y = \dfrac{x^2-10x+25}{x^2-x-20}$

HOLE AT X=5

$y = \dfrac{(x-5)(x-5)}{(x-5)(x+4)}$ *VA: X=-4*

$y = \dfrac{x-5}{x+4}$

X-INT (y=0)
X-5=0 X=5
(5,0)

y-int: (x=0)
$y = \dfrac{(0-5)(0-5)}{(0-5)(0+4)}$
$= \dfrac{25}{-20}$ $\left(0, -\dfrac{5}{4}\right)$

b. $y = \dfrac{(x+4)(x-5)(x+8)}{(x-3)(x+4)(x+1)}$

HOLE: $\boxed{X=-4}$

y-int (x=0)
$y = \dfrac{(0+4)(0-5)(0+8)}{(0-3)(0+4)(0+1)}$
$= \dfrac{4(-5)(8)}{-3(4)(1)}$
$= \dfrac{-40}{-3}$
$y = 13\frac{1}{3}$
$\boxed{\left(0, 13\frac{1}{3}\right)}$

x-int (y=0)
$0 = (x+4)(x-5)(x+8)$
$X+4=0$ $X-5=0$ $X+8=0$
$X=-4$ $X=5$ $X=-8$
$\boxed{(-4,0)\ (5,0)\ (-8,0)}$

VA: $X-3=0$ $X+1=0$
$\boxed{X=3}$ $\boxed{X=-1}$
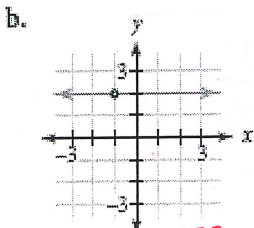
Example 2: First identify the characteristics seen in the graphs below. Then use these characteristics to write the equation for each graph.
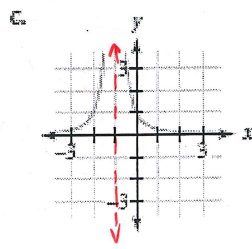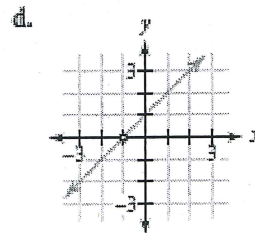
a.

VA: X=-1

$y = \dfrac{1}{x+1}$

b.

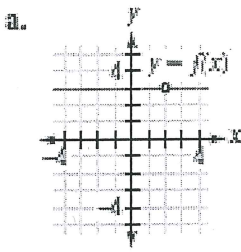
Looks like y=2
HOLE X=-1

$y = \dfrac{2(x+1)}{(x+1)}$

c.

VA: X=-1

$y = \dfrac{1}{(x+1)^2}$

d.

HOLE AT X=-1
Graph looks like y=x+1

$y = \dfrac{(x+1)(x+1)}{(x+1)}$

$y = \dfrac{(x+1)^2}{x+1}$
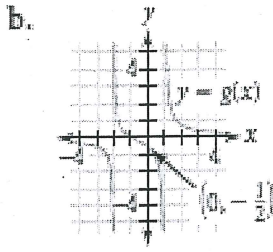
a.

$y = f(x)$

LOOKS LIKE

$y = 3$

HOLE AT $x = 2$

$$y = \frac{3(x-2)}{(x-2)}$$

b.

$y = g(x)$

$\left(0, -\frac{1}{2}\right)$

VA: $x = -2$ $x = 1$

$$y = \frac{}{(x+2)(x-1)}$$

x-int: $x = -1$

$$y = \frac{a(x+1)}{(x+2)(x-1)}$$

y-int $\left(0, -\frac{1}{2}\right)$
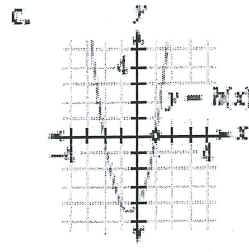
$$-\frac{1}{2} = \frac{a(0+1)}{(0+2)(0-1)}$$

$$-\frac{1}{2} = \frac{1a}{-2}$$

$$\frac{-\frac{1}{2}}{-\frac{1}{2}} = \frac{-\frac{1}{2}a}{-\frac{1}{2}}$$

$$1 = a$$

$$y = \frac{(x+1)}{(x+2)(x-1)}$$

c.

$y = h(x)$

x-int: $(-2,0) \ (1,0)$

HOLE $x = 1$

$$y = \frac{a(x+2)(x-1)(x-1)}{(x-1)}$$

$$y = \frac{a(x+2)(x-1)^2}{x-1}$$

y-int: $(0, -4)$

$$-4 = \frac{a(0+2)(0-1)^2}{(0-1)}$$

$$-4 = \frac{a(2)(1)}{-1}$$

$$\frac{-4}{-2} = \frac{-2a}{-2}$$

$$2 = a$$

$$y = \frac{2(x+2)(x-1)^2}{x-1}$$